NAG Toolbox for MATLAB

f08zb

1 Purpose

f08zb solves a real general Gauss–Markov linear (least-squares) model problem.

2 Syntax

```
[a, b, d, x, y, info] = f08zb(a, b, d, 'm', m, 'n', n, 'p', p)
```

3 Description

f08zb solves the real general Gauss–Markov linear model (GLM) problem

$$\underset{x}{\text{minimize}} \|y\|_2 \qquad \text{subject to} \qquad d = Ax + By$$

where A is an m by n matrix, B is an m by p matrix and d is an m element vector. It is assumed that $n \leq m \leq n+p$, $\text{rank}(A) = n$ and $\text{rank}(E) = m$, where $E = (A \quad B)$. Under these assumptions, the problem has a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of the matrices A and B.

In particular, if the matrix B is square and nonsingular, then the GLM problem is equivalent to the weighted linear least-squares problem

$$\underset{x}{\text{minimize}} \left\|B^{-1}(d - Ax)\right\|_2.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J 1992 Generalized QR factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

2: **b**(**ldb,**$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{p})$

The m by p matrix B.

3: **d**($*$) **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{m})$.

The left-hand side vector d of the GLM equation.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The dimension of the array **d**.

m, the number of rows of the matrices A and B.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $0 \leq \mathbf{n} \leq \mathbf{m}$.

3: **p – int32 scalar**

Default: The second dimension of the array **b**.

p, the number of columns of the matrix B.

Constraint: $\mathbf{p} \geq \mathbf{m} - \mathbf{n}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a is overwritten.

2: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{p})$

B is overwritten.

3: **d($*$) – double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{m})$.

d is overwritten.

4: **x($*$) – double array**

Note: the dimension of the array **x** must be at least $\max(1, \mathbf{n})$.

The solution vector x of the GLM problem.

5: **y($*$) – double array**

Note: the dimension of the array **y** must be at least $\max(1, \mathbf{p})$.

The solution vector y of the GLM problem.

6: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **d**, 9: **x**, 10: **y**, 11: **work**, 12: **lwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$

The upper triangular factor R associated with A in the generalized RQ factorization of the pair (A, B) is singular, so that $\mathrm{rank}(A) < M$; the least squares solution could not be computed.

info $= 2$

The bottom $(N - M)$ by $(N - M)$ part of the upper trapezoidal factor T associated with B in the generalised QR factorization of the pair (A, B) is singular, so that $\mathrm{rank}(AB) < N$; the least squares solutions could not be computed.

7 Accuracy

For an error analysis, see Anderson *et al.* 1992. See also Section 4.6 of Anderson *et al.* 1999.

8 Further Comments

When $p = m \geq n$, the total number of floating-point operations is approximately $\frac{2}{3}(2m^3 - n^3) + 4nm^2$; when $p = m = n$, the total number of floating-point operations is approximately $\frac{14}{3}m^3$.

9 Example

```
a = [-0.57, -1.28, -0.39;
     -1.93, 1.08, -0.31;
     2.3, 0.24, -0.4;
     -0.02, 1.03, -1.43];
b = [0.5, 0, 0, 0;
     0, 1, 0, 0;
     0, 0, 2, 0;
     0, 0, 0, 5];
d = [1.32;
     -4;
     5.52;
     3.24];
[aOut, bOut, dOut, x, y, info] = f08zb(a, b, d)
```

```
aOut =
    3.0562   -0.2694   -0.0232
    0.5322   -1.9623    0.7189
   -0.6343   -0.1120    1.3913
    0.0055    0.2893    0.5605
bOut =
   -1.2131    0.2669   -0.8691    0.6131
   -0.5707    0.7105    1.8876    1.8248
    0.1229    0.2076    2.4417    2.9277
   -0.0373   -0.1437   -0.2765   -2.3759
dOut =
    1.9889
```

```
   -1.0058
   -2.9911
   -0.0094
x =
    1.9889
   -1.0058
   -2.9911
y =
   -0.0006
   -0.0025
   -0.0047
    0.0077
info =
           0
```